**Final report of Digital Signal Processing**



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**Date of Submission:** 2014-12-18

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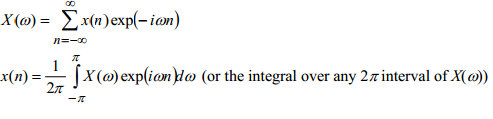
**Fourier transforms discrete time Fourier transform discrete Fourier transform z-transform**

**Introduction**

Here in we describe the relationship between, Discrete Time Fourier Transform (DTFT), and the Discrete Fourier Transform (DFT). Why? The real reason is that the DFT is easily implemented on a computer and is part of every mathematics package, so it would be nice to know how to determine or approximate the DFT and DTFT on a computer.

First, the definitions: ~x n is a periodic function (period N) and x (n) is a non-periodic function

The **discrete-time Fourier transform (DTFT)** is a form of Fourier analysis that is applicable to the uniformly spaced samples of a continuous function.



Of it can be readily calculated via the Discrete Fourier Transform (DFT) (see Sampling the DTFT), which is by far the most common method of modern Fourier analysis.

Both transforms are invertible. The inverse DTFT is the original sampled data sequence. The inverse DFT is a periodic summation of the original sequence.

The Fast Fourier Transform is an algorithm for computing one cycle of the DFT, and its inverse produces one cycle of the inverse DFT

**Definition**

The **discrete-time Fourier transform** (or **DTFT**) of a discrete set of real or complex numbers: *x*[*n*], for all integers *n*, is a Fourier series, which produces a periodic function of a frequency variable. When the frequency variable, ω, has normalized units of radians/sample, the periodicity is 2π, and the Fourier series is**:**

X_{2\pi}(\omega) = \sum_{n=-\infty}^{\infty} x[n] \,e^{-i \omega n}.

The **discrete Fourier transform** (**DFT**) converts a finite list of equally spaced samples of a function into the list of coefficients of a finite combination of complex sinusoids, ordered by their frequencies, that has those same sample values. It can be said to convert the sampled function from its original domain (often time or position along a line) to the frequency domain.

The sequence of **N** complex numbers x_0, x_1, \ldots, x_{N-1} is transformed into an **N**-periodic sequence of complex numbers:

The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.

In digital signal processing, the function is any quantity or signal that varies over time, such as the pressure of a sound wave, a radio signal, or daily temperature readings, sampled over a finite time interval (often defined by a window function). In image processing, the samples can be the values of pixels along a row or column of a raster image.

The **Fourier transform** expresses a function of time (or signal) in terms of the amplitude (and phase) of each of the frequencies hat make it.

This is similar to the way in which a musical chord can be expressed as the amplitude (or loudness) of the notes that make it up. The resulting function, (complex) amplitude that depends on frequency, is called the frequency domainrepresentation of the function.

The term Fourier transform refers both to the operation that associates to a function its frequency domain representation, and to the frequency domain representation itself.

There are several common conventions for defining the Fourier transform \hat{f} of an integrable function f : \mathbb R \rightarrow \mathbb C

In mathematics and signal processing, the **Z-transform** converts a discrete time signal, which is sequence of real or complex numbers, into a complex frequency domain representation.

The Z-transform, like many integral transforms, can be defined as either a one-sided or two-sided transform

**Properties**

One of the most important properties of the DTFT is the convolution property: y[n] = h[n] ∗ x[n]

DTFT ↔ Y (ω) = H (ω) X (ω). This property is useful for analyzing linear systems (and for filter design), and also useful for “on paper” convolutions of two sequencesh[n] and x[n], since if the sequences are simple ones whose DTFTs are known or are easily determined, we can simply multiply the two transforms and then “look up” the inverse transform to get the convolution.

The Fourier transform translates between convolution and multiplication of functions. If *f*(*x*) and *g*(*x*) are integral functions with Fourier transforms \hat{f}(\xi) and \hat{g}(\xi) respectively, then the Fourier transform of the convolution is given by the product of the Fourier transforms \hat{f}(\xi) and \hat{g}(\xi) (under other conventions for the definition of the Fourier transform a constant factor may appear).

The convolution theorem for the discrete-time Fourier transform indicates that a convolution of two infinite sequences can be obtained as the inverse transform of the product of the individual transforms. An important simplification occurs when the sequences are of finite length, **N**. In terms of the DFT and inverse DFT, it can be written as follows**:**

**1. Discreet time Fourier transform**

The Discrete-Time Fourier transform (DTFT) X (ejω ) of sequence x[n] is given by

X (ejω )= ∑x[n]e− jω n

X (ejω ) is called the magnitude function

θ (ω) is called the phase function

Both quantities are again real functions of ω in many applications; the DTFT is called the Fourier spectrum

Likewise: X (ejω ) andθ (ω) are called the magnitude and phase spectrum

To represent a finite energy sequence that is not absolutely summable by a DTFT, it is necessary to consider a mean square.

**2. Discrete Fourier transform**

For a length-N sequence x[n] defined for ≤ n ≤ N− 1 only N samples of its DTFT required which are obtained by uniformly sampling from the definition of DTFT we thus have.

X[k] = X (ejω )ω=2π k / N= ∑x[n] e−j 2π k / N

Note: X[k] is also a length-N sequence in the frequency domain the sequence x [k] is called discrete fourier transform of the sequence x[N].Like the DTFT, the DFT also satisfies a number of properties that are useful in signal processing applications

Some of these properties are essentially identical to those of the DTFT, while some others are somewhat different

**Circular shift sequence:**

This property is analogous to the time- shifting property of the DTFT, but with a subtle difference.

Consider length-N sequences defined for 0≤ n ≤ N – 1, Sample values of such sequences are equal

To zero for values of n < 0 and n ≥ N

If x[n] is such a sequence, then for an arbitrary integer no the shifted sequence

x1[n] =x[n− no ]

is no longer defined for the range 0 ≤ n ≤ N − 1

**Circular convolution:**

This operation is analogous to linear convolution, but with a subtle difference

Consider two length-N sequences, g[n]and h[n], respectively.

Since the operation defined involves two length-N sequences, it is often referred to as an N-point circular convolution, denoted as.

y[n] = g[n] \*h[n]

**Linear convolution using DFT:**

Linear convolution is a key operation in many signal processing applications Since a DFT can be efficiently implemented using FFT algorithms, it is of interest to develop methods for the implementation of linear convolution using the DFT.

**3. z-transform**

The DTFT provides a frequency-domain representation of discrete-time signals and LTI discrete-time systems

• Because of the convergence condition, in many cases, the DTFT of a sequence may not exist

• As a result, it is not possible to make use of such frequency-domain characterization in these cases

A generalization of the DTFT defined by.

X (ejω )= ∑ x[n]e− jω n

Z-transform may exist for many sequences for which the DTFT does not exist ore over, use of z-transform techniques permits simple algebraic manipulations.

Consequently, z-transform has become an important tool in the analysis and design of digital filters

For a given sequence g[n], its z-transformG (z) is defined as

G (z) =∑ g[n] z − n

Where z = Re (z) + jIm (z) is a complex variable

For a given sequence, the set R of values of z for which its z-transform converges is called the region of convergence (ROC).

The z-transforms of the two sequenceαnµ[n]and− αnµ[−n− 1]Only way a unique sequence can be associated with a ztransform is by specifying its ROC

The existence of the DTFT does not always imply the existence of the z-transform.

**4. Fourier Transform**

Fourier transform is defined as continuous.

I F transform1

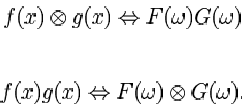
Inverse transform gets rid of freq. components

I F transform2 inverse

In general, Fourier transform is complex

I F transform complex

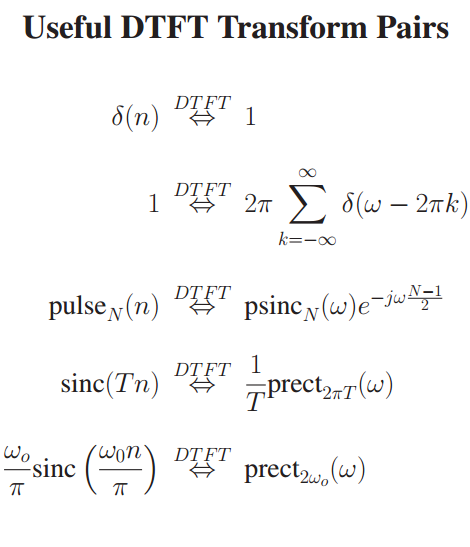
Convolution Property

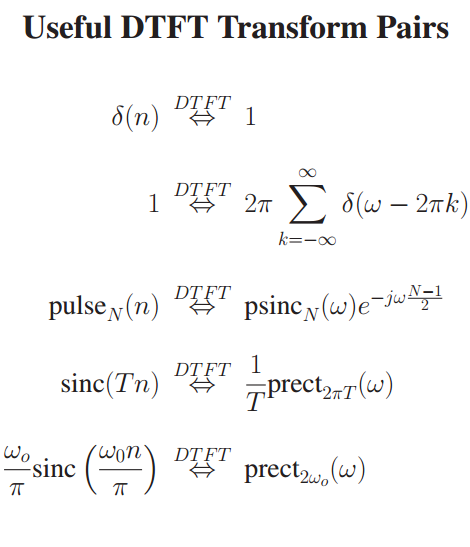


All real world signals are “band limited” That is, they don’t have infinite frequencies nor infinite spatial extend. This is good, otherwise our discrete Fourier copies would collide and alias together. But, what if we still sample too seldom? Even band limited will eventually collide.

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